Project Report  
EEL6503  Spread Spectrum and CDMA  

Performance of Multicarrier DS-CDMA Systems  

Introduction  

Since the radio frequency spectrum is a scarce resource, future wireless radio networks need to make efficient use of the frequency spectrum by providing high capacity in terms of the number of users allowed in the system. As a consequence, new communication technologies are developed to provide more efficient sharing of resources. Such a new scheme combines the multicarrier modulation and Code division multiple access (CDMA).

Actually, there are 2 main types of multicarrier CDMA system which are Multicarrier Code Direct Sequence Division Multiple Access (MC-DS-CDMA) which will be investigated in this project and Multicarrier carrier Multiple Access (MC -CDMA). There is a little difference between these two systems. In this former system, the data sequence multiplied by a spreading sequence modulates $M$ carriers. In the latter system, a spreading sequence is serial-to parallel converted, and each bit of spreading sequence is modulated by a carrier frequency. This implies that the number of carrier must be equal to the number of bits in spreading sequence.

Recently, MC-DS-CDMA systems have received widespread interest. This is due to its effectiveness and robustness in mitigating the effect of multipath fading and in rejecting narrowband interference. In addition, it requires lower chip rate since in a multicarrier DS system with $M$ carriers, the entire bandwidth of the system is divided into $M$ and then each carrier frequency is modulated by a spreading sequence with a chip duration which is $M$ times as long as that of single carrier system. In other words, it requires lower speed, parallel-type of signal processing in contrast to a fast, serial-type of signal processing in a single carrier Rake receiver. This might be helpful for use with a low power consumption device [1]. The purpose of this project is to evaluate performance of MC-DS-CDMA systems over slowly Rayleigh fading channel in the absence of partial band interference.

System Overview  

Figure.1 (a) shows bandlimited wideband direct sequence waveform with a single carrier in frequency domain. The bandwidth ($BW$) of wideband system can be approximated by

$$BW_{\text{single}} = \left(1 + \gamma\right)\frac{1}{T_c}$$

Where $0 < \gamma \leq 1$ and $T_c$ is chip duration of single carrier system. In multicarrier system, we divide bandwidth of single carrier into $M$ sub equal frequency bands and
there is no overlap between each band. Therefore, the bandwidth of each frequency band can be given by

\[ BW_{\text{multicarrier}} = (1 + \gamma) \frac{1}{MT_c} \]

It implies that the chip duration of multicarrier system should be \( MT_c \) and \( M \) is the number of carriers.

Figure 1. (a) psd of wideband single carrier DS waveform. (b) psd of wideband multicarrier DS waveform.

Simulation Model

A. Transmitter

Consider the transmitter shown in Figure 2 where the random binary input sequence of \( k \)th user, \( d_n^{(k)} \in \{-1, 1\} \) is multiplied by a common pseudo-random spreading sequence \( c_n^{(k)} \in \{-1, 1\} \). We assume that there are \( N_c \) chip per symbol and each user has different PN sequence. After passing through the shaping filter, the signal output is modulated by \( M \) carriers (center frequency of subcarrier). We assume that all signal are transmitted with the same energy per bit chip \( E_c \) Then, the transmitted signal of \( k \)th user can be given by

\[ s_m^{(k)}(t) = \sum_{l=1}^{M} \sqrt{E_c} \sum_{n=-\infty}^{\infty} d_n^{(k)} c_n^{(k)} p_{T_c} (t - nT_c) \cos(\omega_c t + \theta) \]
B. PN sequence

In this simulation, gold sequence is used to produce pseudorandom sequence since it gives us good periodic cross correlation properties. In general, gold sequence can be constructed easily by taking a pair of specially selected \( m \)-sequence, called preferred \( m \)-sequence, and forming the modulo-2 sum of two sequences for each of \( L=2^n-1 \) cyclically shifted versions of one sequence relative to other sequence. The \( L \) gold sequences are constructed as shown in Fig.

In this project, the sets of preferred pair of \( m \)-sequence are generated by the primitive polynomials <45> and <67>.

---

**Figure 2.** Block diagram of a multicarrier DS CDMA transmitter.

**Figure 3** Generation of Gold sequence (\( L=31 \))
C. Shaping Filter

In this section, we assume that there is no partial band interference and our signal modulated by each carrier is transmitted over bandlimited channel $W$, where $W \leq (f_{i+1}-f_i)/2$ and $f_i$ is the $i$th carrier frequency. Hence, the chip wave-shaping filter is selected so that it satisfies the Nyquist criterion to guarantee that the spectra do not overlap each other. With $H(f)$ the transfer function of wave-shaping filter.

$$H_{RCF}(f) = |H(f)|^2$$

and

$$\int_{-\infty}^{\infty} |H(f)|^2 df = 1$$

Let $H_T(f)$ be the frequency response of transmitter filter and $H_R(f)$ be the frequency response of receiver filter. Then, the product, $H_T(f).H_R(f)$, is designed as follows:

$$H_{RCF}(f) = |H_T(f)|H_R(f)$$

where $H_{RCF}$ is raised-cosine frequency characteristic which is defined as:

$$H_{RCF}(f) = \begin{cases} Tc & 0 \leq |f| \leq \frac{1-\alpha}{2Tc} \\ Tc/2 \left(1 - \cos \frac{\pi Tc}{\alpha} \left(|f| - \frac{1-\alpha}{2Tc}\right)\right) & 1-\alpha/2Tc \leq |f| \leq 1+\alpha/2Tc \\ 0 & |f| > 1+\alpha/2Tc \end{cases}$$

where $\alpha = $ roll-off factor and frequency resolution factor $(N_f)$ is odd. Since $H_T(f)$ is bandlimited, we will select the sampling frequency $F_s$ to be at least $2/Tc$ and frequency separation $\Delta F = F_s/N_f$. Thus, the impulse response of transmitter and receiver is

$$h_T(n) = \sum_{m=-\left(\frac{N-1}{2}\right)}^{\left(\frac{N-1}{2}\right)/2} \sqrt{H_{RCF}(\frac{mF_s}{N})} e^{j2\pi m n / N} \quad n = 0, \pm 1, \ldots, \pm \frac{N-1}{2}$$

E. Channel Model

In order to provide comprehensive performance analysis for single carrier RAKE system and multicarrier system, we are going to classify the channels impulse response into 2 models.

For single carrier system, the channel is assumed to be a frequency selective Rayleigh fading channel with coherence bandwidth $(\Delta f)_c$. Recall that the channel is classified as frequency selective if the bandwidth of transmitted signal $(BW)$ is large when compared with $(\Delta f)_c$, namely
Then, at the receiver, we would receive several copies for transmitted signal with different delays. With this model, the number of resolvable paths of channel, $L_1$, is given by

$$L_1 = \left\lfloor \frac{BW}{(\Delta f)_c} \right\rfloor + 1$$

where the coherence bandwidth is related to delay spread, $T_m$, by

$$(\Delta f)_c \approx \frac{1}{T_m}$$

The channel impulse response can be modeled as in Figure 4. where $\beta$ is zero mean complex Gaussian random variable.

For multicarrier system, we are going to choose the number of carriers to achieve the following requirement [1].

- Each sub frequency band of muticarrier system has no selectivity, i.e., $T_m/(MT_c) \leq 1$.

To satisfy the first requirement, we need to choose $M$ equal to $L_1$

- All sub frequency bands are subject to independent fading which implies that $BW_m \geq (\Delta f)_c$.

To satisfy this requirement, we need to choose $\gamma \geq T_c/T_m$

With this assumption above, the channel impulse response for $i$th subcarrier can be given by

$$c(t) = \sum_{l=0}^{L-1} \beta \delta(t - lT_c)$$

where $\lambda_i$ is i.i.d., zero mean complex Gaussian random variable.
F. Receiver

For single carrier system, the transmitted signal undergoes frequency selective and multiple transmission paths. Therefore, we employ Rake receiver to improve the performance of signal. Here, we assume that we have perfect timing and channel estimation. Rake receiver can be implemented as in Figure 5.

\[ r(t) \]

\[ \sqrt{2} \cos(\alpha \cdot t + \theta) \]

\[ \sum_{n=0}^{N-1} c_n(t) \]

\[ \text{sgn(Re(.))} \]

\[ \beta_0^* \]

\[ \beta_1^* \]

\[ \beta_{L-1}^* \]

Figure 5. Rake receiver for single carrier system.

For multicarrier system, we employ Maximum Ratio Combining (MRC) method as receiver. In maximum ratio combining, the signals in all branches are individually weighted to provide the optimal SNR at the output. Recall that the output SNR is maximized when the signals in each diversity branches weighted by their own envelopes can be given by

\[ Z = \sum_{i=1}^{M} r_i Z_i \]

where \( r_i \) represents the instantaneous envelopes of signals received at each diversity branch. The SNR per bit at the output of the maximum ratio combining can be written as:

\[ \gamma_b = \sum_{i=1}^{M} \gamma_k = \frac{E}{N_0} \sum_{i=1}^{M} \alpha_i \]

where \( \alpha_i \) is Rayleigh fade envelope.

Figure 6. shows that signals in all branches are demodulated by M carriers. The outputs of correlators are combined according to the manner described above:
**Theoretical Results**

**Single-User Case:**

In single carrier system, the conditional signal to noise ratio (SNR) at the output of Rake receiver can be expressed as the following: [1]

\[
\rho_s = \frac{2N_0E_c}{N_0} \sum_{l=0}^{L-1} \alpha_{l,j}^2
\]

Whereas, the conditional signal to noise ratio of multicarrier maximum ratio combining receiver can be written as: [1]

\[
\rho_m = \frac{2MNE_c}{N_0} \sum_{j=1}^{M} \alpha_{i,j}^2
\]

**Multi-User Case:**

The conditional SNR of single carrier Rake receiver can be obtained as: [1]

\[
\rho_m = \left\{ \frac{K-1}{2N_1} \left( 1 - \frac{\gamma}{4} \right) + \frac{N_0}{2N_1E_c} \right\}^{-1} \sum_{l=0}^{L-1} \alpha_{i,j}^2
\]

The conditional SNR of multicarrier receiver can be written as: [1]

\[
\rho_m = \left\{ \frac{K-1}{2MN} \left( 1 - \frac{\gamma}{4} \right) + \frac{N_0}{2MNE_c} \right\}^{-1} \frac{1}{M} \sum_{j=1}^{M} \alpha_{i,j}^2
\]
Simulation Results

Case I

To calibrate the system model, the channel is first assumed to be Additive white Gaussian Noise (AWGN).

![The performance of Multicarrier over AWGN channel](image)

**Figure 7.** The performance of MC-DS-CDMA over AWGN channel.

Figure 7. shows the performance of MC-DS-CDMA over AGWN channel when the number of carrier is set to be 4. It can be seen that the performance of MS-CDMA over AWGN channel is the same as that of BPSK case. This implies that transmitter and receiver module function properly.

Case-II (Single User Case)

Simulation parameters:
- The number of users (K) = 1
- The length of spreading sequence=31
- The number of bits (N) = 8000
- The number of iteration = 5

In single carrier system, we assume that signal undergoes to a frequency selective channel and the number of paths is set to be equal to 2. For this system, Rake receiver is employed to improve the performance. However, the performance can get better if multicarrier system is employed. Moreover, it can be seen that the performance of MC-DS-CDMA is improved when we increase the number of carriers. In Figure. 8 BER curve is shown for different numbers of carriers.
The Number of User (K) = 1

Figure 8. BER versus E_b/N_o of MC-DS-CDMA and single carrier Rake system for K=1.

Note that due to limitation of the length of gold sequence, we cannot compare a multicarrier system with a single carrier Rake receiver having the same bandwidth.

Case-III (Multi-User Case)

Figure 9. BER versus E_b/N_o for multi-user
Simulation parameters

- The number of bits = 8000
- The length of spreading sequence=31
- The number of iterations=3
- The number of carriers=4

Figure 9. shows BER versus $E_b/N_0$ for synchronous multi-user case when the number of carriers is equal to 4. It can be seen that the more users, the poorer performance. This is because the multiple access interference (MAI) between users is major factor in limiting the performance.

**Conclusion and Future Work**

In CDMA system, the disadvantage of wideband CDMA system with single carrier system is that the signal can easily experience to a frequency selective channel since the bandwidth of signal is large when compared to the coherence bandwidth. With this reason, the performance of system becomes undesirable even though we employ RAKE receiver to improve the performance. However, in multicarrier CDMA system I have studied, we can obtain the narrowband DS waveform by properly dividing the available bandwidth to many subchannels so that the bandwidth of each subchannel is less than the coherence bandwidth and each subband is subject to independent fading. By doing that, each subband of multicarrier system has no selectivity. Therefore, this type of signaling has more desirable property of narrowband interference suppression and robustness to the fade channel.

In this project, I get to understand the basic structure of MC-DS-CDMA. By developing some simulations. I know the effect of different parameters to overall performance.

The future work is listed as follows:

- Apply other types of spreading sequences to compare both single carrier Rake system and multicarrier system with the same available bandwidth.
- Develop the new kinds of receivers to improve performance of system such as multi-user detector.
- Study the effect of timing and channel estimation.
References

Appendix

Matlab code

% finding m-sequence
function [seq] = max_length(connection);
connection = fliplr(connection);

% number of registers
m = length(connection)-1;

% length of sequences
L = 2^m-1;

% initial condition
register = [zeros(1,m-1),1];
initial = register;
seq(1) = register(1);
for i=2:L,
    for j=1:m-1,
        new_reg_cont(j) = register(j+1);
    end;
    new_reg_cont(m) = register(1);
    for k=2:m,
        new_reg_cont(m) = new_reg_cont(m)+register(k)*connection(k);
        new_reg_cont(m) = mod(new_reg_cont(m),2);
    end;
    register = new_reg_cont;
    seq(i) = register(1);
    if (initial==register)
        disp('the connection is not valid to generate maximum length sequences');
        break;
    end;
end;

% find other all sequences
for a =2:L,
    seq(a,:) = wshift('1D',seq(a-1,:),1);
end;

% generating Gold sequence
function [gold]= gold
u=max_length([1 0 0 1 0 1]);
v=max_length([1 1 0 1 1 1]);
gold=mod(u+v,2);

% Raised cosine filter
function [y] = RCF(f,alpha,T,fc);
if (abs(f-fc) > ((1+alpha)/(2*T))),
y = 0;
elseif (abs(f-fc) > ((1-alpha)/(2*T))),
y = (T/2)*(1+cos((pi*T/alpha)*(abs(f-fc)-(1-alpha)/(2*T))));
else
    y = T;
end;
end;
%Raised cosine filter

function [g_T] = Filter(N,T,\alpha,\text{Fs},\text{fc})

n = -(N-1)/2:1:(N-1)/2;
for k = 1:length(n),
g_T(k) = 0;
for m = -(N-1)/2:1:(N-1)/2,
g_T(k) = g_T(k) + sqrt(RCF(Fs*m/(N),\alpha,T,\text{fc})) * \exp((j*2*pi*m*n(k)/N));
echo off;
end;
g_T = real(g_T);
g_T = g_T/max(abs(g_T));

%transmitter

function [\text{trans}] = transmitter(\text{data},t,\text{Nsample},\text{Nc},\text{pn},\text{M},\text{fc})

Nf=21;
\alpha=0.5;
T=1;
Tc=(T/\text{Nc});
Fs=\text{Nsample}/Tc;
%To save time we can assume impulse response of filter to be 1
for no=1:Nusers,
repeated_data(no,:) = kron(data(no),ones(1,Nc));
dsss(no,:) = repeated_data(no,:).*pn;
[ht] = Filter(Nf,Tc,\alpha,\text{Fs},0);
sampled_dsss(no,:) = kron(dsss(no,:),ones(1,\text{Nsample}));
a(no,:) = conv(sampled_dsss(no,:),ht);
filter_output = a(1+(Nf-1)/2:length(a)-(Nf-1)/2);
for d=1:M,
mod(d,:) = sqrt(2)*filter_output(no,:).*\cos(2*\pi*\text{fc}(d)*t);
end;
end;
%transmitted_signal
\text{trans} = sum(mod,1);

%Receiver for multicarrier system(MRC)
function [\text{z}] = receiver(\text{rec},h,t,\text{Nsample},\text{Nc},\text{pn},\text{M},\text{fc})

Nf=21;
\alpha=0.5;
T=1;
Tc=(T/\text{Nc});
Fs=\text{Nsample}/Tc;

%Multiplying multicarrier M
for b=1:M,
demod(b,:) = sqrt(2)*rec.*\cos(2*\pi*\text{fc}(b)*t);
[ht] = Filter(Nf,Tc,\alpha,\text{Fs},0);
aa=conv(demod(b,:),ht);
filter_output(b,:) = aa(1+(Nf-1)/2:length(aa)-(Nf-1)/2);
%sampling
for k=1:Nc,
    rec1(b,k)=filter_output(b,((k-1)*Nsample+1));
end;
%despread
    temp(b)=conj(h(b))*sum(rec1(b,:).*pn(1,:));
end;
    y(a)=sum(real(temp));
z=sum(y);
%Decision

%Receiver for signal carrier (Rake receiver)

function [z] = receiverf(rec,h,t,Nsample,Nc,pn,M,fc)

Nf=11;
alpha=0.5;
T=0.004;
Tc=(T/Nc);
Fs=Nsample/Tc;

    temp=[zeros(1,1),rec];
    for i=1:2,
        r(i,:)=wshift('1D',temp,i-1);
        branch(i,:)=conj(h(i))*r(i,:);
    end;
symbol_range=1+(M-1):length(branch)-(M-1);
rake_branch = branch(:,symbol_range);

    for b=1:M,
        demod(b,:) = sqrt(2)*rake_branch(a,:).*cos(2*pi*fc(b)*t);
        [ht] = Filter(Nf,Tc,alpha,Fs,0);
        filter_output(b,:)=aa(1+(Nf-1)/2:length(aa)-(Nf-1)/2);
        %sampling
        for k=1:Nc,
            rec1(b,k)=filter_output(b,((k-1)*Nsample+1));
        end;
        %despread
            temp(b)=sum(rec1(b,:).*pn);
        end;
    y(a)=sum(real(temp));
z=sum(y);
%Decision

function [avp] = test7(snr_in_dB,M,pn,h)

    snr=10^(snr_in_dB/10);
Nusers=3;
N=1000; Nc=31;
T=1; Tc=(T/Nc); %chip duration of MC-DS-CDMA
Nsample=1;
ts=Tc/Nsample; fs=1/ts;
%To save run time, we can select fc to be [1:M]*10000;
%To ensure that bandwidth of subcarrier doesn't overlap each other.
for i=1:M,
    fc(i)=10000+(1/(2*Tc));
end;
error=0;
iteration=3;

14
for n=1:iteration,
    for i=1:N,
        % Sampling time
        t=[(i-1)*T:ts:i*T-ts];
        for no=1:Nusers,
            data(no)=sign(rand-0.5);
        end;
        % Transmitter
        trans=transmitter(data,t,Nsample,Nc,pn,M,fc,Nusers);
        E=sum(trans.^2);
        sigma=sqrt(E/snr);
        % For single carrier
        % a=conv(trans,h);
        % e=length(a);
        % noise=sigma*(randn(1,e)+j*randn(1,e));
        % rec=a+noise;
        % For multicarrier system
        for path=1:M,
            rand('state',sum(100*clock));
            noise=sigma*(randn(1,Nc)+j*randn(1,Nc));
            rec(path,:)=h(path)*trans+noise;
        end;

        % Receiver
        z=receiver(rec,h,t,Nsample,Nc,pn,M,fc);
        % decision
        if z<0
            decision=-1;
        else
            decision=1;
        end;
        if decision~=data(1)
            error=error+1;
        end;
    end;
    p(n)=error/N;
end;
avp=sum(p(n))/iteration;

% plot function
function plot2
    snr_in_dB=0:1:10;
    sequence=gold;
    M=4;
    Nusers=3;
    for no=1:Nusers;
        pn(no,:)=(-1).^sequence(no,:);
    end;
    h=zeros(1,M);
    for path=1:M
        h(path)=randn+j*randn;
end;
for i=1:length(snr_in_dB),
    err_prob1(i) = test(snr_in_dB(i),M,pn,h);
    echo off;
end;
semilogy(snr_in_dB,err_prob1,'R*-');
hold on
grid